4.1	Matrix Operations
Objectives	1. Add and subtract matrices, multiply by a scalar, and solve matrix equations. 2. Use matrices in real life applications.
Key Terms	Matrix Dimensions Entries (Elements) Scalar Equal Matrices
Comparing Matrices	Are the matrices equal? $\begin{bmatrix} -6 & \frac{1}{2} \\ \frac{3}{5} & 1 \end{bmatrix}, \begin{bmatrix} -6 & 0.5 \\ 0.6 & 1 \end{bmatrix}$ $\qquad$ $\begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$
Adding and Subtracting Matrices	To add/subtract matrices, the matrices must have the same dimension. $\begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 5 \\ -1 & -3 \end{bmatrix}$ $\qquad$ $\begin{bmatrix} -2 & 3 & 5 \\ 1 & 0 & -4 \end{bmatrix} - \begin{bmatrix} 5 & -8 & 6 \\ -4 & 6 & -2 \end{bmatrix}$

Multiplying a Matrix by a Scalar	Multiply each entry by the scalar number. $2\begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix}$ $\frac{1}{2}\begin{bmatrix} 3 & 8 & 0 \\ \frac{2}{3} & -10 & -2 \\ \frac{1}{3} & 7 & 1 \end{bmatrix}$
Perform the Indicated Operations.	$3\begin{bmatrix} -5 & 1 & 6 \\ 0 & -8 & -2 \end{bmatrix} - 2\begin{bmatrix} 4 & -1 & -7 \\ 3 & 5 & -5 \end{bmatrix}$
Solving a Matrix Equation	Find the value of x and y. $2\left(\begin{bmatrix} -2 & 3 \\ x & 5 \end{bmatrix} + \begin{bmatrix} y & 7 \\ -4 & 10 \end{bmatrix}\right) = \begin{bmatrix} -10 & 20 \\ 10 & 30 \end{bmatrix}$
The population estimate for Town A in 2009 was 36,315 males and 33,352 females and for Town B the population estimate was 6,452 males and 6,633 females. **In 2010 the population estimate for Town A was 36,998 males and 34,208 females for Town B it was 6,434 males and 6, 672 females.**	Organize the information into a matrix, then find the change in population by subtracting the matrices
Properties of Matrices	Associative (A + B) + C = A + (B + C) Commutative A + B = B + A Distribution of a Scalar c(A + B) = cA + cB c(A - B) = cA - cB

4.2	Multiplying Matrices
Objectives	1. Multiply two matrices. 2. Use matrix multiplication in real life application.
Multiplying Matrices	To multiply, the number of **columns** of the first matrix must **EQUAL** the number of **rows** of the **second** matrix A · B = AB m x n n x p m x p
Multiply, if possible.	A = $\begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 5 & -2 \end{bmatrix}$ B = $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$ Find AB: Find BA:

Using Matrix Operations	A= $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ B = $\begin{bmatrix} -2 & 0 \\ 4 & 2 \end{bmatrix}$ C = $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$ Find AB: Find A(B + C):
	Properties of Matrix Multiplication Associative Property of Matrix Mult. A(BC) = (AB)C Left Distributive Property A(B + C) = AB + AC Right Distributive Property (A + B)C = AC + BC Associative Property of Scalar Mult. c(AB) = (cA)B = A(cB)
The women's softball and men's baseball teams need new equipment. The women's team needs 16 bats, 42 balls, and 16 uniforms. The men's team needs 14 bats, 50 balls and 15 uniforms. Bats cost $85, balls cost $4 and uniforms cost $50. Write matrices and multiply to find total cost.	

4.3	Determinants and Cramer's Rule
Objectives	1. Evaluate determinants of 2 x 2 and 3 x 3 matrices. 2. Use Cramer's Rule to solve systems of linear equations.
Key Terms	Determinant of a 2 x 2 Matrix Determinant of a 3 x 3 Matrix
Find the determinate of the 2 x 2 matrix.	$A = \begin{bmatrix} 6 & 5 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -7 & 11 \\ -7 & 2 \end{bmatrix}$
Find the determinant of the 3 x 3 matrix.	$A = \begin{bmatrix} -2 & 0 & 1 \\ 4 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 0 & 1 \\ 0 & 8 & 9 \\ 0 & 5 & 2 \end{bmatrix}$
Area of a Triangle with the vertices of (x_1, y_1), (x_2, y_2), (x_3, y_3)	$\text{Area} = \pm\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ **Find the area of the triangle with the vertices of (1, 2) , (3, -4) and (5, 3)**

Use Cramer's Rule to solve the system of equations.	2x – 3y = 2 -3x + 6y = 0
Use Cramer's Rule to solve the system of equations.	4x – 3y = 18 8x – 7y = 34
Cramer's Rule for a 2x2 System	ax + by = e cx + dy = f $x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A}$ $\qquad$ $y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$

Use Cramer's Rule to solve the system of equations.	$x - 4y + 2z = 2$ $-x + 6y - z = 1$ $2x - 5y + z = 4$
Cramer's Rule for a 3x3 System	$ax + by + cz = j$ $dx + ey + fz = k$ $gx + hy + iz = l$ $x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}$ $y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}$ $z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}$

4.4	**Identity and Inverse Matrices**
Objectives	1. Find and use inverse matrices. 2. Use inverse matrices in real life applications.
Key Terms	Identity Matrix (2x2) and (3x3) Inverse Matrix A^{-1}
The inverse of matrix A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$	$A^{-1} = \frac{1}{\lvert A \rvert}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ **provided ad – bc ≠ 0**
Find the inverse of the matrix.	$A = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ -5 & -1 \end{bmatrix}$ $C = \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix}$

Solve the matrix equation. **Find the inverse of matrix A and then multiply both sides of the equation by A^{-1} on the LEFT.**	$$\underset{A}{\begin{bmatrix} 5 & 1 \\ -4 & -1 \end{bmatrix}} X = \underset{B}{\begin{bmatrix} 11 & 13 \\ -9 & -10 \end{bmatrix}}$$
Solve the matrix equation	$$\underset{A}{\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}} X = \underset{B}{\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}}$$

4.5	**Solving Systems Using Inverse Matrices**
Objectives	1. Solve systems of linear equations using inverse matrices. 2. Use systems of linear equations to solve real life problems.
Key Terms	Matrix of Variables Matrix of Constants.
	Let A be the matrix of the coefficients **Let X be the matrix of the variables** **Let B be the matrix of the constants** $AX = B$ $A^{-1}AX = A^{-1}B$ **To solve, multiply by A^{-1} on the left.** $X = A^{-1}B$
Write the system of linear equations as a matrix equation and then solve.	$-3x + 4y = 5$ $2x - y = -10$

Write the system of linear equations as a matrix equation and then solve.	$-x - 2y = 3$ $2x + 8y = 1$