9.1	**Inverse and Joint Variation**
Objectives	1. Write and use inverse variation models. 2. Write and use joint variation models.
Key Terms	Direct Variation Inverse Variation Constant of Variation Joint Variation
Classifying Direct or Inverse Variation	1. Solve the equation for y. 2. If it is $y = kx$ then it is **direct variation** and if it is $y = \frac{k}{x}$ then it is **inverse variation**. **Equation** $2 = xy$ $x = \frac{y}{7}$ $y = x - 2$
The variables x and y vary inversely. When x = 3, y = 2. Write an equation that relates x and y. Then, find y when x = 8.	

The variables x and y vary inversely. When x = 1.5, y = 6. Write an equation that relates x and y. Then, find y when x = 0.5.

Tell whether the data shows an inverse variation. If it does, write a model for the relationship between x and y.

x	5	10	15	20	25
y	21	10.5	7	5.25	4.2

Joint Variation

Write an equation for the given relationship.

y varies jointly with x and z.

y varies inversely as the with the cube of x.

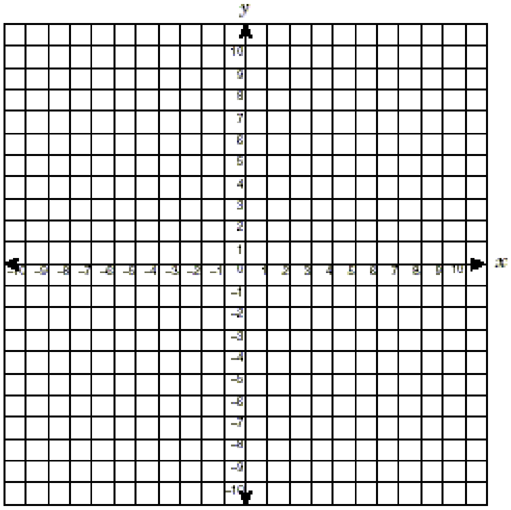
y varies directly with x.

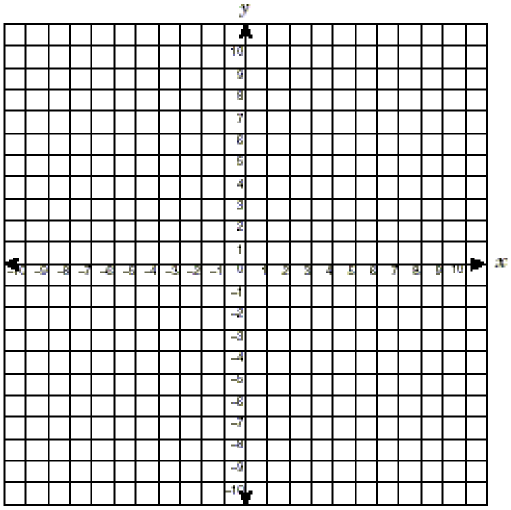
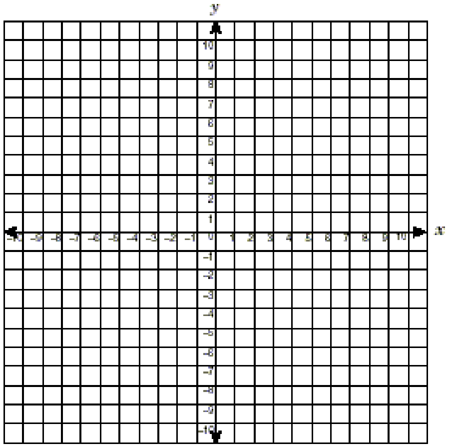
y varies inversely with x.

z varies jointly with x, y and w.

z varies directly with x and inversely with y.

The force F needed to loosen a bolt with a wrench varies inversely with the length L of the handle. Write an equation relating F and L, given that 250 lbs. of force must be exerted to loosen a bolt when using a wrench with a 6in. long handle. How much force must be exerted when using a wrench with a 24 in. long handle?

9.2	**Graphing Simple Rational Functions**
Objective	1. Graph simple rational functions.
Key Terms	Rational Function
	Properties of Hyperbolas - The graph has a **vertical** and **horizontal asymptote**. - **Domain** and **Range** are all real numbers except where the asymptotes are. - The graph has two symmetrical parts called **branches**.
	Rational Function in the form of: $\boldsymbol{y = \frac{a}{x-h} + k}$ The vertical asymptote is x = h. The horizontal asymptote is y = k. Graph the asymptotes first then set up table of values to graph the branches.
Graphing Rational Functions.	Graph: $y = \frac{3}{x-2} + 1$  Vertical Asymptote ________________ Horizontal Asymptote ________________ Domain ______________ Range ________________

	Graph: $y = \frac{4}{x} - 2$  Vertical Asymptote ______________ Horizontal Asymptote ______________ Domain ______________ Range ______________
	All functions of the form. $$\boldsymbol{y = \frac{ax+b}{cx+d}}$$ are rational functions. **Vertical Asymptote** is what makes the x value of the denominator zero. **Horizontal Asymptote** is $\boldsymbol{y = \frac{a}{c}}$
Graphing Rational Functions.	Graph: $y = \frac{9x+1}{3x-2}$  Vertical Asymptote ________________ Horizontal Asymptote _________________ Domain _________________ Range ________________

9.3	**Graphing General Rational Functions**
Objective	1. Graph general rational functions.
Key Terms	Rational Functions

Graphing Rational Functions

$$\boldsymbol{f(x)=\frac{p(x)}{q(x)}=\frac{a_m x^m+a_{m-1}x^{m-1}+\ \ldots.+a_1x^1+a_0}{b_n x^n+b_{n-1}x^{n-1}+\ \ldots.+b_1x^1+b_0}}$$

m is the degree of the numerator.
n is the degree of the denominator**.**

1. The **x-intercepts** of the graph are the real zeros of the numerator, p(x). Set the numerator equal to zero to find them.

2. The graph of the function has **vertical asymptote** at each real zero of the denominator, q(x). Set the denominator equal to zero to find it**.**

3. The graph has at most one **horizontal asymptote**:

   If m < n, the horizontal asymptote is __________.

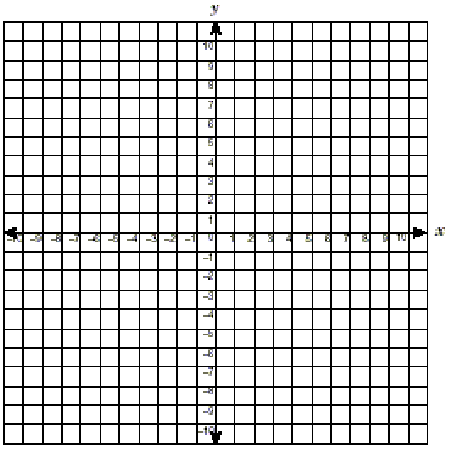
   If m = n, the line ______________ is the horizontal asymptote.

   If m >n, the graph has ________ horizontal asymptote. The graph has an end behavior the same as the graph of

   _________.

Graphing a Rational Function (m < n)

Graph: $y = \frac{6}{x^2+3}$



m = __________

n = __________

Horizontal Asymptote

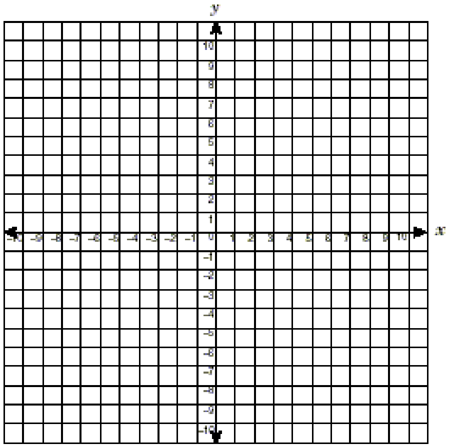
____

Vertical Asymptote

____

Graphing Rational Function (m = n)

Graph: $y = \frac{x^2-4}{x^2-1}$



m = __________

n = __________

Horizontal Asymptote

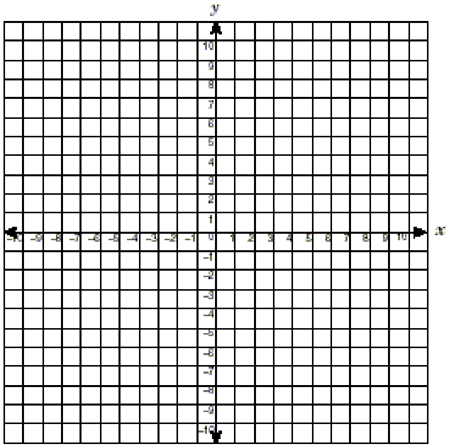
____

Vertical Asymptote

____

Graphing Rational Functions (m > n)

Graph: $y = \frac{x^2-5x+4}{x-5}$



m = __________

n = __________

Horizontal Asymptote

____

Vertical Asymptote

____

9.4	**Multiple and Divide Rational Expressions**
Objective	1. Multiply and divide rational expressions.
Key Terms	Simplified Form
Simplifying Rational Expressions	Let a, b, and c be non-zero numbers or variable expressions. The following property applies: $\frac{ac}{bc} = \frac{a}{b}$ Divide out common factor of c. **To simplify a rational expression:** 1. Factor the numerator and denominator. 2. Divide out any factors common to both the numerator and denominator.
Simplify the expression.	$\frac{x^2-2x-3}{x^2-8}$ $\frac{x^2+2x-15}{x^2+6x+5}$ $\frac{2x^3+32x}{x^2+8x+16}$
Multiplying Rational Expressions	1. Factor, if possible. 2. Multiply as you would numerical fractions. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ 3. Divide out any factors common to both the numerator and denominator.
Multiply, simplify the result.	$\frac{6x^2y^3}{2x^3y^2} \cdot \frac{10x^3y^4}{18y^2}$ $\frac{16x^3}{5y^9} \cdot \frac{x^5y^8}{80x^3y}$

Multiply, simplify the result.	$\frac{3x^2+6x}{x^2-4x}\cdot\frac{x-4}{x^2+9x+14}$ $\frac{x-3}{xy^2}\cdot\frac{4x^2y^3}{x^2+x-12}$ $\frac{2x}{x^3-1}\cdot(x^2+x+1)$
Dividing Rational Expressions	1. Factor if possible. 2. Divide as you would numerical fractions, which is to multiply the first expression by the **reciprocal** of the second expression then simplify. $\boldsymbol{\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}}$
Divide and Simplify.	$\frac{3}{4x-8}\div\frac{x^2+3x}{x^2+x-6}$ $\frac{20x^2+x-1}{7x^3+x}\div\frac{4x+1}{x}$

Divide and Simplify.	$\frac{8x^2+10x-3}{4x^2} \div (4x^2 - x)$ $\frac{x^3-4x}{2} \div (x^4 - 16)$
Combined Operations **Perform the indicated operations and simplify the result.**	$\frac{x}{x-2} \cdot (2x+3) \div \frac{x^2-9}{x-2}$

9.5	Addition, Subtraction and Complex Fractions
Objectives	1. Add and subtract rational expressions. 2. Simplify complex fractions.
Key Term	Complex Fraction
Adding or Subtracting with like denominators	1. Add or subtract numerators and place the result over the like denominator. 2. Simplify.
Add or subtract	$\frac{3}{2x} - \frac{7}{2x}$ $\frac{x}{x^2+5x} + \frac{5}{x^2+5x}$
Adding or Subtracting with unlike denominators	1. Factor the denominator. 2. Find the Least Common Denominator (LCD) of the denominators. 3. Rewrite each expression as an equivalent rational expression. 4. Perform the indicated operation and simplify.
Add or subtract, then simplify.	$\frac{7}{5x} + \frac{8}{3x}$ LCD=________

Add or subtract, then simplify.	$\frac{5}{6x^2}+\frac{x}{4x^2-12x}$ LCD=____________ $\frac{x}{x-4}-\frac{6}{x+3}$ LCD=____________ $\frac{2x}{x^2-1}-\frac{x+2}{x^2+2x+1}$ LCD=____________

Simplifying Complex Fractions	1. Rewrite complex fraction as division. 2. Total the two quantities in parenthesis. 3. Multiply the first expression by the **reciprocal** of the second expression, then simplify.
Simplify	$\frac{\frac{x-2}{3x+1}}{\frac{1}{x}-\frac{2}{3x+1}}$ $\frac{\frac{x}{5}+4}{8+\frac{1}{x}}$ $\frac{25-\frac{1}{x^2}}{\frac{1}{5x^2-x}}$

9.6	**Solving Rational Equations**
Objectives	1. Solve rational equations. 2. Use rational equations to solve real life problems.
Solving Rational Equations	1. Multiply each term on both sides of the equation by the LCD of all the terms. 2. Simplify. 3. Solve the resulting equation. 4. Check for extraneous solutions.
Solve.	$\frac{3}{x}-\frac{1}{2}=\frac{12}{x}$ $\frac{x}{x-3}=2-\frac{2}{x-3}$ $4-\frac{5}{x+1}=\frac{5x}{x+1}$

Solve.	$\frac{2}{x+1}+\frac{x}{x-1}=\frac{2}{x^2-1}$
Solve by cross multiplying.	Cross multiplying can only be used if each side of the equation is a single rational expression. $\frac{x-4}{x}=\frac{6}{x^3-3x}$
So far in the basketball season you have made 12 free throws out of 20 you have attempted, for a free throw percentage of 60%. How many consecutive free throws would you have to make to raise your percentage to 80%?	