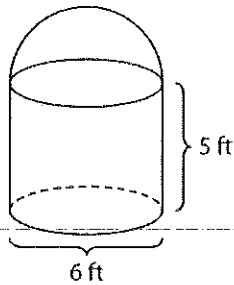


## Midterm Review Integrated 3 Short Response 2016-2017

1. A company sells several sizes of the same design of trash cans. The trash cans consist of a cylinder and a hemisphere. The smallest size trash can has the dimensions shown.



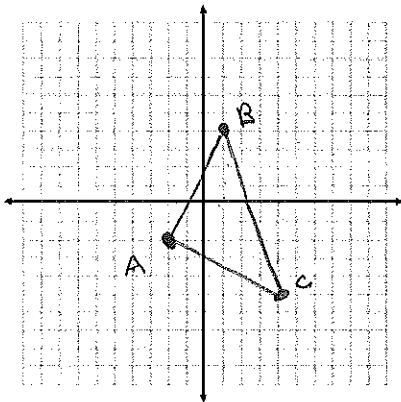
$$V_{\text{cyl}} = \pi (3)^2 (5) = 45\pi$$

$$V_{\text{sph}} = \frac{2}{3}\pi (3)^3 = 18\pi$$

What is the volume of the trash can?

$$45\pi + 18\pi = 63\pi \text{ ft}^3$$

2. Is the triangle with vertices  $A(-2, -2)$ ,  $B(1, 4)$ , and  $C(4, -5)$  isosceles? Explain/Show.



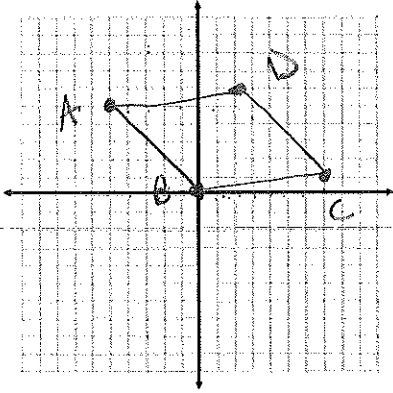
$$\overline{AB} = \sqrt{(1-(-2))^2 + (4-(-2))^2} = \sqrt{45}$$

$$\overline{AC} = \sqrt{(4-(-2))^2 + (-5-(-2))^2} = \sqrt{45}$$

$$\overline{BC} = \sqrt{(4-1)^2 + (-5-4)^2} = \sqrt{90}$$

Yes  $AB = AC$

3. Three vertices of quadrilateral  $ABCD$  are  $A(-5, 5)$ ,  $B(0, 0)$ , and  $C(7, 1)$ . What are the coordinates of the fourth point such that the quadrilateral is a rhombus?



$$D(2, 6)$$

$$AB = \sqrt{50}$$

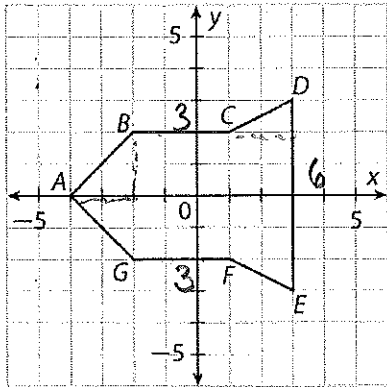
$$DC = \sqrt{(7-2)^2 + (1-6)^2} = \sqrt{50}$$

$$AD = \sqrt{(-5-2)^2 + (5-6)^2} = \sqrt{50}$$

$$BC = \sqrt{(7)^2 + (1)^2} = \sqrt{50}$$

$$D(2, 6)$$

4. What is the perimeter of the figure?



$$DE = 6$$

$$BC = 3$$

$$GF = 3$$

$$AB = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$AG = \sqrt{8}$$

$$CD = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$FE = \sqrt{5}$$

$$P = 6 + 3 + 3 + \sqrt{8} + \sqrt{8} + \sqrt{5} + \sqrt{5}$$

$$P = 22.13 \text{ units}$$

5. A right cone has a slant length of 22 centimeters and a radius of 14 centimeters. What is the surface area of the cone?

$$S = \pi r l + \pi r^2$$

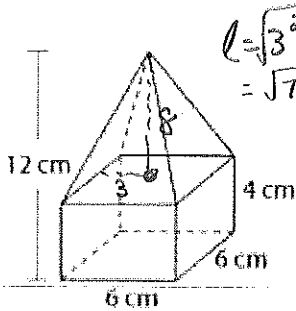
$$= \pi (14)(22) + \pi (14)^2$$

$$= \pi \cdot 308 + 196\pi$$

$$= 504\pi \text{ cm}^2$$

$$\approx 1583.36 \text{ cm}^2$$

6. What is the surface area of the composite figure of a square pyramid on top of a cube?



$$l = \sqrt{3^2 + 8^2} \\ = \sqrt{73}$$

$$S_{\text{prism}} = Ph + B \\ = 24(4) + (36) = 132$$

$$L_{\text{pyr}} = \frac{1}{2}Pl \\ = \frac{1}{2}(24)(\sqrt{73}) \\ = 12\sqrt{73}$$

$$S = 12\sqrt{73} + 132$$

$$S \approx 234.5 \text{ cm}^2$$

7. The dimensions of the triangle are decreased by a scale factor of  $\frac{1}{3}$ . What is the area of the resulting triangle compared to the original?

$$\text{Area} = \frac{1}{9} \text{ original}$$

8. The density of a steel ball is 16 grams per cubic centimeter. What is the mass of a steel ball with radius 4 centimeters?

$$D = \frac{M}{V_{\text{sphere}}}$$

$$16 = \frac{M}{\frac{256}{3}\pi}$$

$$V = \frac{4}{3}\pi r^3 \\ = \frac{4}{3}\pi(4)^3 \\ = \frac{256}{3}\pi$$

$$16\left(\frac{256}{3}\pi\right) = M$$

$$M = 4289.3 \text{ grams}$$

9. The territory of a town can be modeled by a rectangle with dimensions 12 miles and 7 miles. If the town has a population of 28,500, what is its population density? Round to the nearest person.

$$\frac{\text{pop}}{\text{area}} = \frac{28,500}{(12 \cdot 7)} = \frac{28,500}{84}$$

$$339 \text{ people/mi}^2$$

10. Use the Binomial Theorem to expand  $(x + 6)^4$ .

$$x^4 + 4x^3(6) + 6x^2(6)^2 + 4x(6)^3 + (6)^4$$

$$\underline{x^4 + 24x^3 + 216x^2 + 864x + 1296}$$

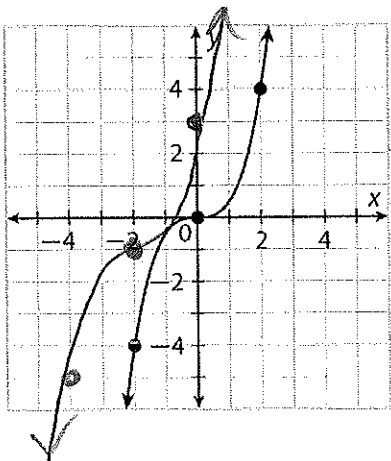
11. Use synthetic division to determine all factors of  $p(x) = x^3 - 4x^2 - 25x + 100$ , given that  $x - 4$  is a factor.

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -25 & 100 \\ & & 4 & 0 & -100 \\ \hline & 1 & 0 & -25 & 0 \end{array}$$

$$x^2 - 25 \\ (x+5)(x-5)$$

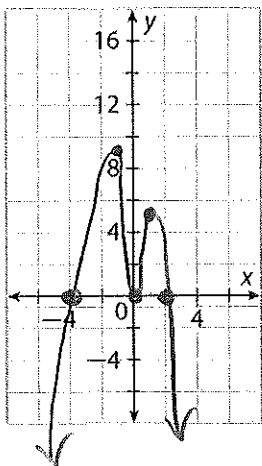
$$\underline{(x-4)(x+5)(x-5)}$$

12. Draw the graph if the function shown below is translated 2 units left and 1 unit down. Draw and label the reference points on the curve that you draw.



$$\begin{aligned} (2, 4) &\rightarrow (0, 3) \\ (0, 0) &\rightarrow (-2, -1) \\ (-2, -4) &\rightarrow (-4, -5) \end{aligned}$$

13. Sketch the graph of
- $f(x) = -x^2(x+4)(x-2)$

End Behavior

$x^4$  even degr. negative  
 As  $x \rightarrow +\infty$ ,  $f(x) \rightarrow -\infty$   
 As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow -\infty$

x-Int

0, -4, 2  
 tangent

Extra Points

(-5, -175) (-1, 9) (3, -63)  
 (-2, 32) (1, -5)

Write the simplest polynomial function with the given zeros.

14. 4,
- $-3i$

$$(x-4)(x-3i)(x+3i)$$

$$(x-4)(x^2-9i^2)$$

$$(x-4)(x^2+9)$$

$$x^3 + 9x - 4x^2 - 36$$

$$\underline{x^3 - 4x^2 + 9x - 36}$$

Solve the polynomial equation by finding all real roots.

- 15.
- $g(x) = x^3 - x^2 - 10x - 8$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & & -1 & 2 & 8 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$

$$x^2 - 2x - 8$$

$$(x-4)(x+2)$$

$$\underline{x = -1, 4, -2}$$

