Algebra/Trigonometry Section 3.4b Name:________________________

Sketch the graph of the quadratic function without using a graphing utility. Identify the vertex and x-intercepts.

1. $f(x) = x^2 + 2x + 1$

Find the quadratic function that has the indicated vertex and whose graph passes through the given point.

2. Vertex: $\left(\frac{5}{2}, -\frac{3}{4}\right)$ Point: $(-2, 4)$

Word problem

3. The height y (in feet) of a ball thrown by a child is $y = -\frac{1}{12}x^2 + 2x + 4$ where x is the horizontal distance (in feet) from the point at which the ball is thrown.

   a. How high is the ball when it leaves the child's hand? (Hint: Find y when $x = 0$)
   b. What is the maximum height of the ball?
   c. How far from the child does the ball strike the ground?

Use long division to divide.

4. $(4x^3 - 7x^2 - 11x + 5) \div (4x + 5)$

Use synthetic division to divide.

5. $(3x^3 - 16x^2 - 72) \div (x - 6)$

Use synthetic division to show that x is a solution of the third-degree polynomial equation, and use the result to factor the polynomial completely. List all the real zeros of the function.

6. $x^3 - 28x - 48 = 0$ $\quad x = -4$
7. $x^3 - 3x^2 + 2 = 0$ $\quad x = 1 \pm \sqrt{3}$

(a) Verify the given factors of $f(x)$, (b) find the remaining factors of $f(x)$, (c) use your results to write the complete factorization of $f(x)$, (d) list all real zeros of $f(x)$.

8. $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$
   factors: $(x - 5)(x + 4)$

(a) List the possible rational zeros of $f(x)$, (b) determine all the real zeros of f.

9. $f(x) = x^3 + x^2 - 4x - 4$
10. $f(x) = 4x^3 - 12x^2 - x + 15$

Find a polynomial function with integer coefficients that has the given zeros.

11. $1, 5i, -5i$

Use the given zero to find all the zeros of the function.

12. $f(x) = 2x^3 + 3x^2 + 50x + 75$; zero: $5i$
13. $f(x) = x^3 - 7x^2 - x + 87$; zero: $5 + 2i$

Use Descartes's Rule of Signs to determine the possible number of positive and negative zeros of the function.

14. $g(x) = 5x^5 + 10x$
15. $f(x) = 3x^3 + 2x^2 + x + 3$

Word problem

16. A company that manufactures bicycles estimates that the profit for selling a particular model is $P = -45x^3 + 2500x^2 - 275{,}000$; if $0 \leq x \leq 50$ where P is the profit (in dollars) and x is the advertising expense (in tens of thousands of dollars). Using this model, find the smaller of two advertising amounts that yield a profit of $800,000.